DISC Course on Modeling and Control of Hybrid Systems

Homework for Chapter 1 and 2

Deadline: Friday 8th of July 2022

1. (**4 pt**) Select a system in your own field of research or interests that can be considered as a hybrid system (and that is not yet discussed in the lecture notes). Give a short description of the system, and describe the system (or a part of it) as a hybrid automaton.

2. Consider the Generalized Transition System (GTS) definition on the lecture slides.

   i. (**2 pt**) Recast the definition of a continuous time differential inclusion in the GTS formalism. **Hint:** Consider carefully how to encode time in the model.

   ii. (**2 pt**) Capture now a Discrete Time Markov Decision Process in the GTS formalism. Think carefully about what should be the state-set of the system.

3. Consider a water tank of diameter d as in Figure 1. The tank has an outlet valve V at a height h_v (letting water leave the tank), and a feeding tap T at the top (providing water to fill the tank). Assume the time needed to completely open or close the outlet valve is negligible, and also consider the distance between the top and bottom of the valve opening negligible. The tank is open at the top, which implies that if the water level exceeds the height of the tank H the tank will overflow.

   Denote by $h(t)$ (in m) the level of the water in the tank at time t. The diameter of the tank is d (also in m). Let $\alpha(t)$ be the inflow rate (in m^3/s) provided by the tap at time t. The outflow rate $\omega(t)$ (in m^3/s) through the valve V is linearly proportional to the volume of water above the valve[1].

   i. (**2 pt**) Derive a continuous-time PWA model describing the evolution of the water level h while the valve V is open.

   Consider now a situation in which the output valve V switches state from open to close, and vice versa, at discrete times (not necessarily equidistant) $\{t_k\}$.

   ii. (**2 pt**) Derive a hybrid automaton capturing the evolution of $h(t)$ in this new situation.

   Assume now that $h_v = 0$, i.e. the outlet is at the bottom of the tank, and that the input $\alpha(t)$ is a piecewise constant signal only changing value at the same time instants t_k, i.e. $\alpha(t) = \alpha_k, \;\; \forall\, t \in [t_k, t_{k+1})$. Furthermore, assume now that the outflow is constant[2] $\omega(t) = \beta$.

   Consider first the case in which $t_{k+1} - t_k = T$, $\forall k \in \mathbb{N}$, i.e. periodic sampling.

[1] For simplicity of the expressions assume $\omega(t)$ is equal to the volume of water above the outlet valve

[2] This could be achieved by placing a pump.

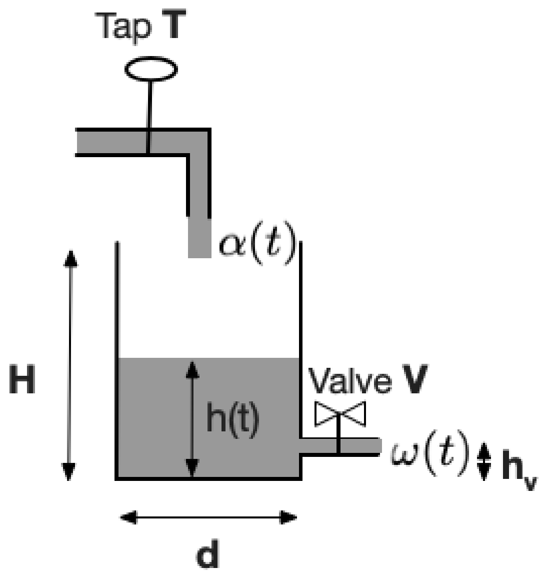

Figure 1: A water tank with an outlet valve and a feeding tap.

iii. (**3 pt**) Derive an MMPS model for the discrete time system with state $x(k)$ representing the water level height at the instants of switching of the valve V, i.e., $x(k) = h(t_k)$.

iv. (**3 pt**) Rewrite the resulting model of the previous step as an MLD model.

Let us study now the case when $t_{k+1} - t_k$ is not constant, that is: aperiodic switching and sampling.

v. (**2 pt**) Can you write the discrete-time system now as an MMPS system? Argue your answer. If your answer is *no*, provide possible re-interpretations of the system allowing you to rewrite it as an MMPS system. (**Hint:** You may consider the time between sampling instances as an additional input $s_k = t_{k+1} - t_k$.)