Lecture 5: Hybrid Systems & Control

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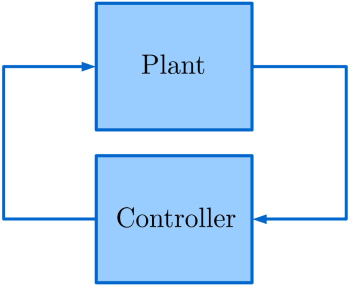
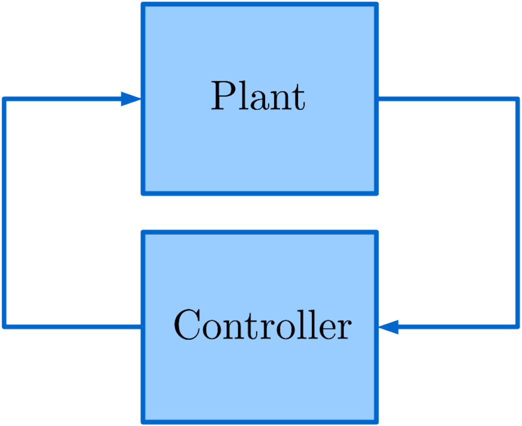
Introduction: what we study in this lecture



$$\begin{cases} \dot{x} \in F(x) & x \in C \\ x^+ \in G(x) & x \in D \end{cases} \qquad (\mathcal{H})$$

Introduction: when does this happen?



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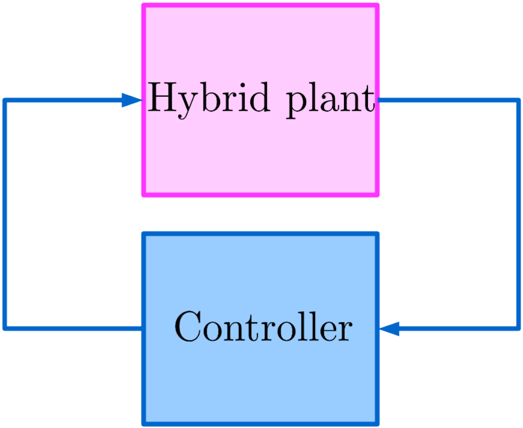

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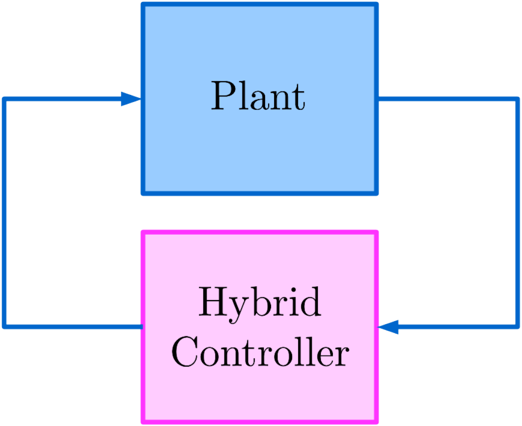

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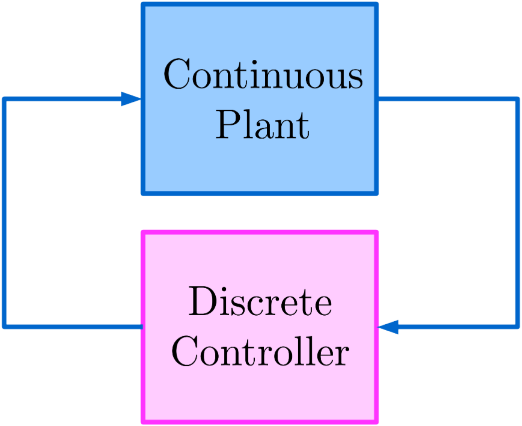

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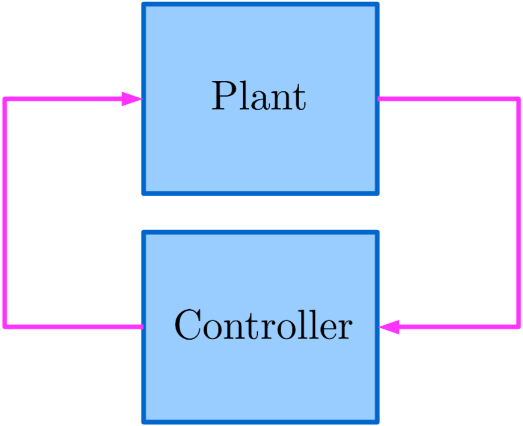


Introduction: presentation style

Much shorter and much less technical than the two previous lectures

We go through each of these categories and present a sample of techniques at a high level.

Far from being an exhaustive view of the field

List of references at the end.

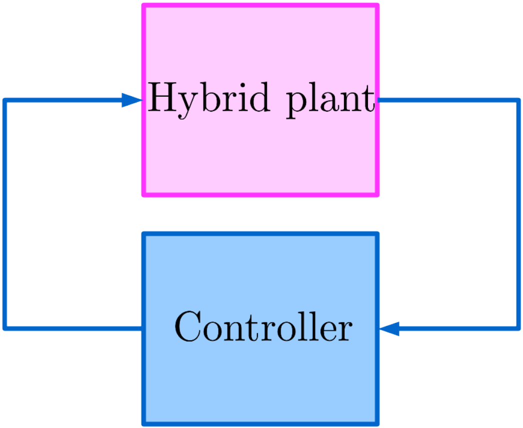
Many of these techniques have not been developed with the hybrid formalism we saw in the previous lectures

Overview

1. Introduction
2. Hybrid plant
3. Hybrid controller
4. Hybrid implementation
5. Discussions
6. Summary

Overview

Hybrid plant: set-up



Hybrid plant: model & objective

Hybrid plant

$$\left\{ \begin{array}{lll} \dot{x}_p & \in & F_p(x_p, u) & (x_p, u) \in C_p \\ x_p^+ & \in & G_p(x_p, u) & (x_p, u) \in D_p, \end{array} \right. \qquad (\mathcal{H}_c)$$

where

- x_p is the plant state,
- u is the control input.

Objective

To design a controller to stabilize a set for $\mathcal{H}_c$.

Hybrid plant: model & objective

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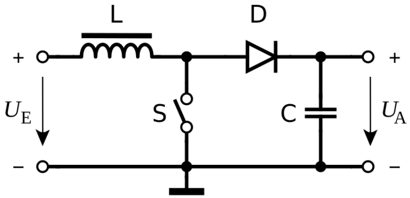


(Source Wikimedia)

Objective

To design a controller to stabilize a set for $\mathcal{H}_c$.

Hybrid plant: switched control

Switched systems

$$\dot{x} = f_\sigma(x, u), \tag{SW}$$

where

- x is the state,
- σ is the switching signal, which may be used for control,
- u is the control input.

We can model SW as $\mathcal{H}$, as we briefly saw.

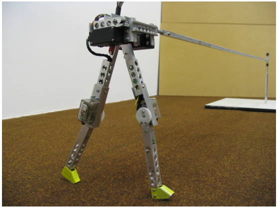
With no doubt, one of the most studied hybrid control problems.

Various approaches are available in the literature.

General idea: (to switch) to make a Lyapunov function decrease "overall" along solutions.

$\rightarrow$ not easy to construct such a Lyapunov function $\rightarrow$ (average) dwell-time conditions.

Hybrid plant: mechanical systems with impact



(Source Wikimedia)

Largely studied in the literature due to its numerous applications

Challenge: to deal with limit cycle, special type of closed attractor.

Most results not developed within the hybrid formalism.

Hybrid plant: control Lyapunov function

To prove stability, we typically use a Lyapunov function

Control Lyapunov function (CLF) are functions, which can be used to construct control laws to enforce the Lyapunov conditions

Consider the differential equation

$$\dot{x} = f(x, u),$$

we say that V is a CLF with respect to closed set $\mathcal{A} \subset \mathbb{R}^n$ for this system if there exist $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ and ρ positive definite such that:

- for all $x \in \mathbb{R}^n$, $\alpha_1(|x|_\mathcal{A}) \leq V(x) \leq \alpha_2(|x|_\mathcal{A})$,
- for all $x \in \mathbb{R}^n$, there exists $u \in \mathbb{R}^m$ such that

$$\langle \nabla V(x), f(x, u) \rangle \leq -\rho(|x|_\mathcal{A}).$$

Concept extended to hybrid systems.

Not common technique, largely underdeveloped in my opinion.

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Hybrid plant: backstepping

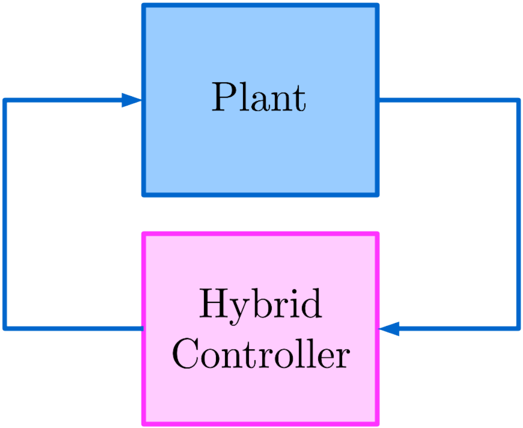
Backstepping is a popular nonlinear control technique for differential equations of the form (strict feedback)

$$\begin{array}{rcl} \dot{x}_1 & = & f_1(x_1) + g(x_1)x_2 \\ \dot{x}_1 & = & u. \end{array}$$

Backstepping has been proposed for a class of hybrid systems.

Overview

Hybrid controller: set-up



Hybrid controller: motivation

Continuous-time plant model

$$\dot{x} = f(x, u) \qquad \text{(CT)}$$

(we could consider a discrete-time plant model, but this is less standard)

Recall: when CT is

- linear, various explicit control techniques are available (pole placement, LQR control, tracking control etc.),
- nonlinear, no general explicit methodology $\rightarrow$ solutions for classes of systems.

Why a hybrid controller?

- to improve performance of continuous-time feedbacks,
- to overcome fundamental limitations of continuous-time feedbacks,
- to ease the controller design.

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Hybrid controller: Brockett integrator

Brockett integrator

$$\begin{array}{rcl} \dot{x}_1 & = & u_1 \\ \dot{x}_2 & = & u_2 \\ \dot{x}_3 & = & x_2 u_1 - x_1 u_2 \end{array} \quad (1)$$

The origin is not globally stabilizable by a continuous feedback law

Mathematical curiosity? → wheeled mobile robot, induction motor with high-current loops.

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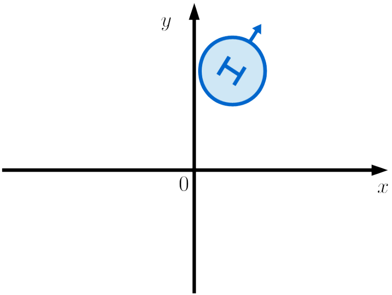

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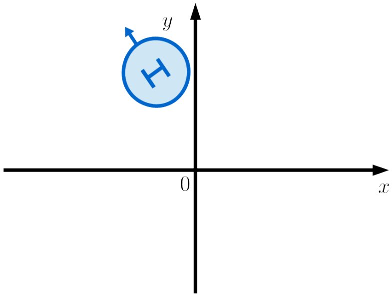

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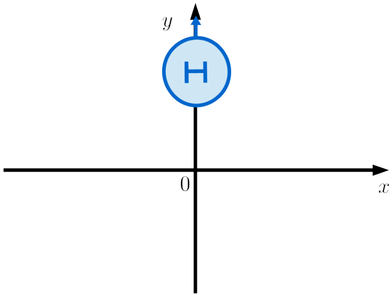

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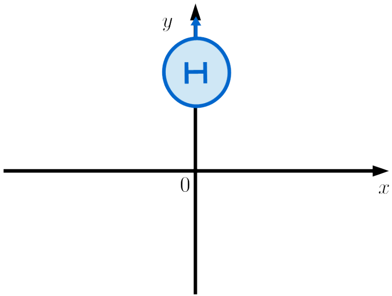


Possible to globally asymptotically stabilize with discontinuous/hybrid feedbacks.

Hybrid controller: reset control

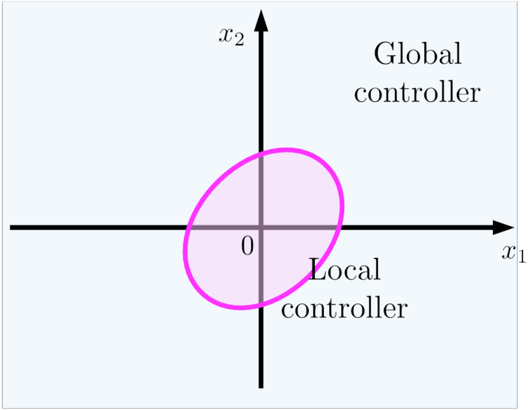
Mostly, but not exclusively, for linear time-invariant systems

Dynamic controller, like PI (proportional-integral)

Idea: to suitably reset the state of the controller to improve performances.

Can significantly improve the system response in terms of overshoot and transient time.

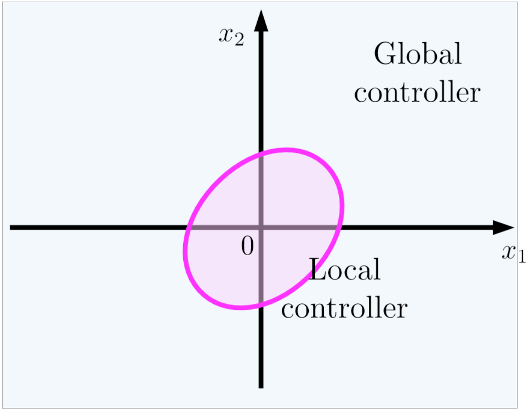
Hybrid controller: uniting control



State- and output-feedback solutions

Also solutions for hybrid plant (and hybrid controller)

Hybrid controller: uniting control



State- and output-feedback solutions

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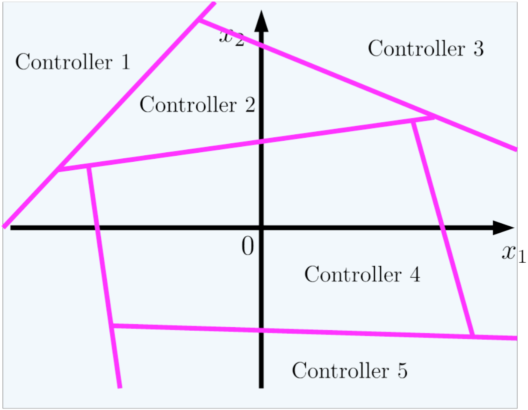
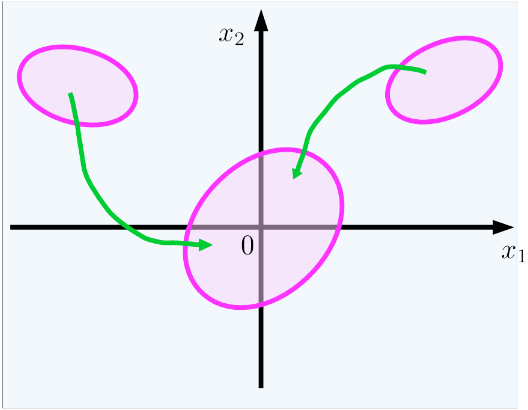
Hybrid controller: patchy control Lyapunov functions (CLF)



To cover the state space with a family of “local” control Lyapunov functions

To derive a hybrid control strategy

Hybrid controller: “throw-and-catch”



Robust UGpAS guarantees

Hybrid controller: supervisory control

Notion of supervisory control well-established in discrete-event systems.

Here, I refer to works initiated by A.S. Morse and his co-authors (J. Hespanha, D. Liberzon, C. De Persis etc).

Consider

$$\dot{x} = f(x, \theta, u)$$

where $\theta \in \mathbb{R}^p$ is a vector of **unknown** parameters.

Objective: to asymptotically stabilize $x = 0$ (not necessarily to estimate θ).
→ traditional problem in adaptive control.

Difficult when f depends nonlinearly in θ

Lack of uniform stability properties a priori.

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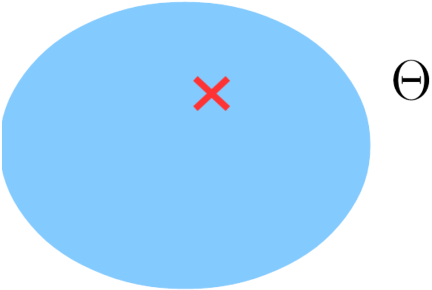
Hybrid controller: supervisory control

- Suppose $\theta \in \Theta$, where Θ is known bounded set
- Discretize Θ with N points
- Design N associated controllers
- Associate a state-observer to each controller
- Selection criterion + apply the considered control law



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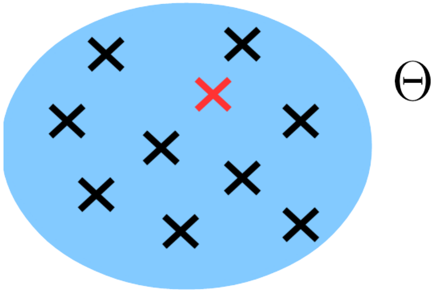

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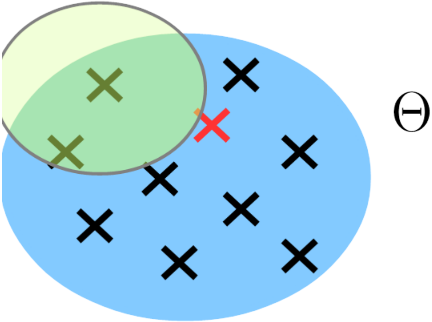

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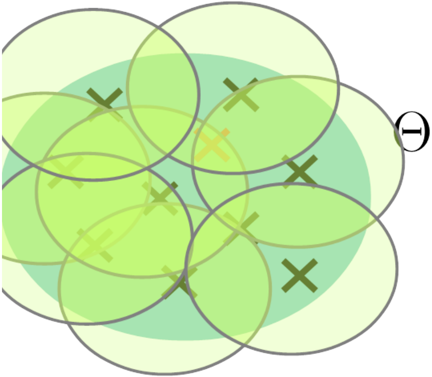

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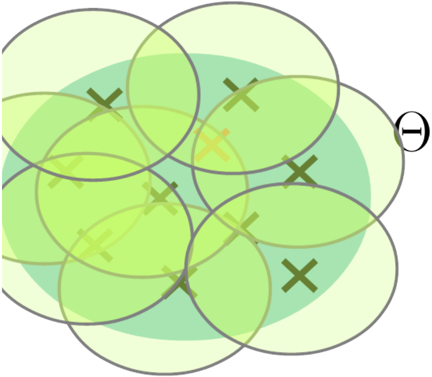

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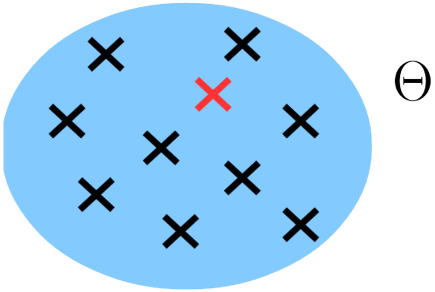

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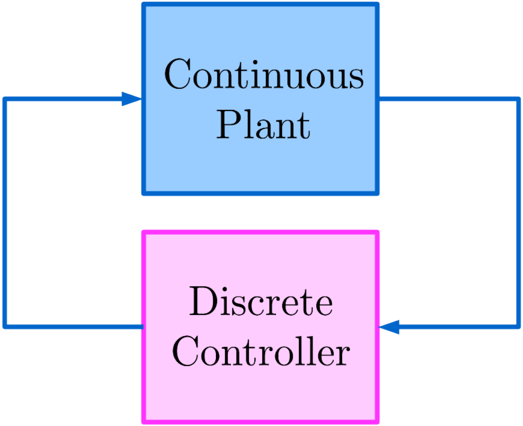
Robust stability guarantees (no persistency of excitation)

Overview

Hybrid implementation: set-up



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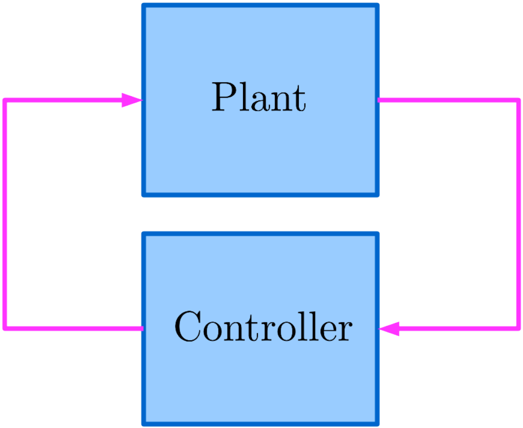
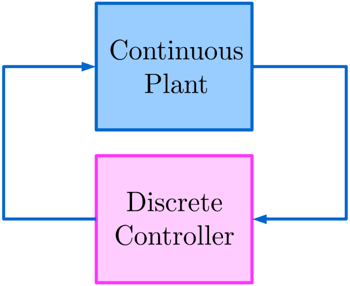


Hybrid implementation: sampled-data control



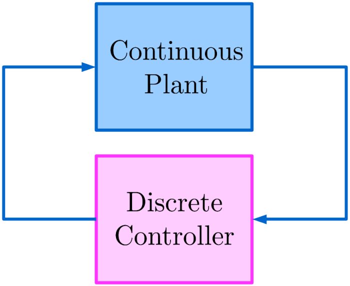
Why to model it as a hybrid system? Why not to model everything in discrete-time?

- To take into account the inter-sampling behaviour
- To cope with varying sampling periods

$$\dot{\tau} = 1 \quad \tau \in [0, T_{\max}], \qquad \tau^{+} = 0 \quad \tau \in [\varepsilon, T]$$

  where $0 < \varepsilon \leq T$.
- Very useful for nonlinear systems for which discretization is tricky.
- Can be used to compute explicit bounds on the sampling period.

Hybrid implementation: sampled-data control



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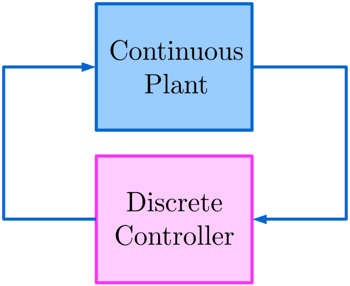

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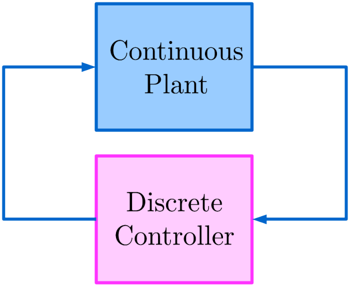

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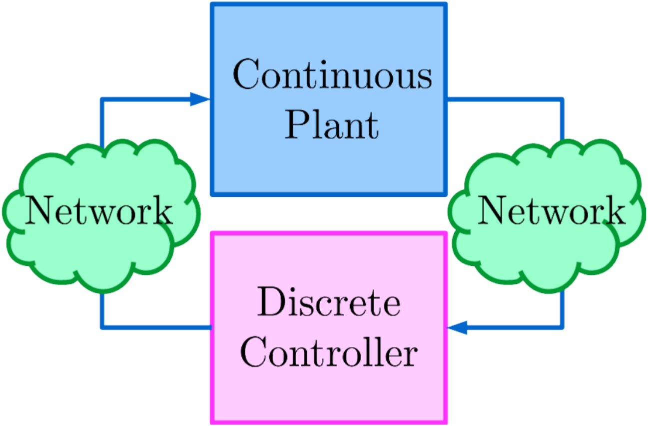
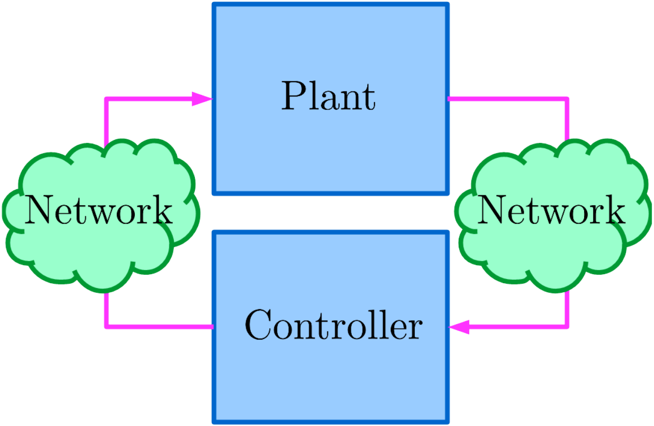
Hybrid implementation: networked control systems



Hybrid implementation: networked control systems



Hybrid implementation: networked control systems

Network effects:

- Aperiodic data sampling,
- Scheduling protocols
- Quantization

$$\hat{y} = q(y), \qquad \text{e.g.} \quad y(t) = 2.127, \ \hat{y}(t) = q(y(t)) = 2.1$$

- Packet loss,
- Time-varying delays.

All these phenomena can be modeled as a hybrid system using the formalism we saw.

Various results available in the literature:

- point stabilization,
- robust stabilization,
- tracking control,
- observer design.

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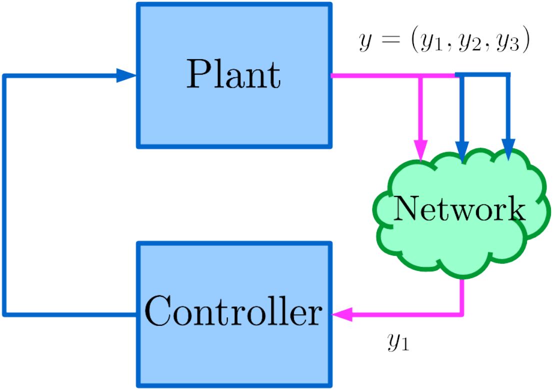

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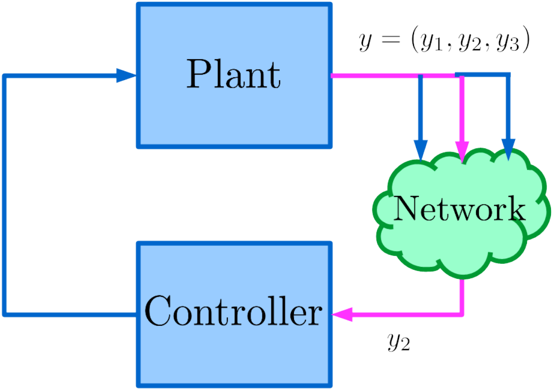

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Hybrid implementation: event-triggered control

Traditionally, transmissions depend on time-triggered clocks, like

$$\dot{\tau} = 1 \quad \tau \in [0, T_{\mathsf{max}}], \qquad \tau^+ = 0 \quad \tau \in [\varepsilon, T]$$

Alternative: to adapt transmissions to the state of the plant
→ to reduce transmissions over the network

Hybrid implementation: event-triggered control

To transmit only when some criterion is positive, like

$$\Gamma(x, \hat{x}) \geq 0,$$

where

- Γ takes scalar values,
- $\hat{x}$ is the sampled version of x

Typical example

$$\text{Transmit when } \Gamma(x(t), x(t_j)) = |x(t) - x(t_j)| \geq c,$$

where $c > 0$.

Hybrid implementation: event-triggered control

We derive

$$C = \{(x,\hat{x}) : \Gamma(x,\hat{x}) \leq 0\} \quad D = \{(x,\hat{x}) : \Gamma(x,\hat{x}) \geq 0\}$$

Challenge: to define the controller and the triggering criterion Γ to ensure

- stability properties,
- to avoid Zeno phenomenon
- to ensure the existence of a strictly positive time between any two transmissions.

Hybrid controller: symbolic control

Consider

$$\dot{x} = f(x, u) \tag{NL}$$

Discretize in time and in space NL (abstraction)

Design of a control strategy

Application to NL

See Manuel's lecture.

Overview

Discussions

What about performance/robust properties?

What about optimal control?

What about tracking control or output regulation?

→ Not easy when the plant solution and the reference trajectory do not jump simultaneously but some results exist

What about observer design?
(switched systems, networked control systems, sampled-data observers, supervisory approach, uniting observers, etc.)

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Overview

Summary

- When control leads to hybrid system.

- Several scenarios and a sample of associated results

Summary: some references

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Summary: some references

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