DISC Course on Modeling and Control of Hybrid Systems

Homework for Chapter 1 and 2

1. (**4 pt**) Select a system in your own field of research or interests that can be considered as a hybrid system (and that is not yet discussed in the lecture notes). Give a short description of the system, and describe the system (or a part of it) as a hybrid automaton.

2. In the lecture notes the formal definition of a system proposed by Sontag is presented in Definition 1.4.1. For linear time-driven systems the lecture notes already indicate how the differential or difference equation for the state can be fitted into Sontag's framework.

   i. (**2 pt**) Recast the definition of a *deterministic* automaton into Sontag's framework. Consider both the case when the time set $\mathcal{T}$ equals $\mathbb{R}$ and the case when $\mathcal{T}$ equals $\mathbb{N}$ (Hint: see also footnote 3 on the lecture notes). Please note that the expression for the map[1] ϕ_{Sontag} does not necessarily have to be in closed analytical form, but may also be of algorithmic nature or in an implicit form.

   ii. (**2 pt**) Can you encapsulate *non-deterministic* automata into the Sontag framework? Discuss your answer.

   Consider now the Generalized Transition System (GTS) definition on the lecture slides.

   iii. (**2 pt**) Recast the definition of a continuous time differential equation in the GTS formalism. **Hint:** Consider carefully how to encode time in the model.

   iv. (**1 pt**) Discuss the differences/similarities of *Sontag's machines* vs *Generalized Transition Systems*. Is one more expressive than the other?

3. Consider a data processing system, a *server*, receiving a data flow at its input and dispatching the data at its output. Think of this server as a queue in which incoming data traffic is stored and after deciding the best route the data is pushed out of the queue. We assume that the server has a maximum service speed of β bits/s, and a maximum queue length L. If the queue is full and new data arrives to the server the new data is discarded.

   To model the incoming traffic and outgoing traffic one can use *cumulative data flows*: $R : \mathbb{N}_0 \to \mathbb{N}_0{}^2$. Cumulative data flows are monotonically increasing functions satisfying $R(0) = 0$ and represent the total (accumulated) amount of bits of information $R(t)$ that an information source has produced up to that point in time t.

   If we denote by $R^*(t)$ the cumulative data flow at the output of the server, one can represent the backlog, i.e. the amount of bits stored in the queue awaiting to be serviced, as $b(t) = R(t) - R^*(t)$. Note that $R^*(t)$ is the amount of bits that the server has dispatched at time t, and thus $R^*(t) < R(t)$ if $R(t) - R(t-1) > 0$. In other words, the server dispatches

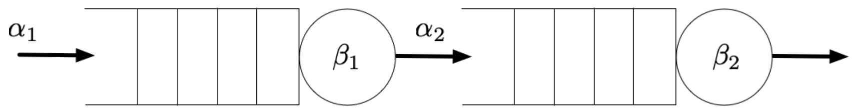

Figure 1: System with two servers.

data that is already in the queue, and thus at time t bits that arrive to the system cannot be immediately dispatched at time t.

Consider a data flow with a arrival rate $\alpha(t) := R(t) - R(t-1)$ going through two servers in a series cascade as in Figure 1.

i. (**2 pt**) Derive an MMPS model for the evolution of the backlogs of the system (the lengths of the two queues).

ii. (**2 pt**) Can you rewrite the MMPS model as a PWA model? If it is possible, provide the PWA model.

iii. (**2 pt**) Describe the system as a Hybrid automaton.

iv. (**1 pt**) What type of state-space does this system have? Think about what would be the state set X if you were to model the system as a generalized transition system.

Consider now a system with only one server.

v. (**1 pt**) If the queue has no maximum length, i.e. $L = \infty$, under which conditions could you rewrite the model as an MLD system?

vi. (**1 pt**) Consider again the system with maximum queue length $L < \infty$ and provide an MLD model of the system.

[1]To indicate the difference between the map ϕ of Sontag's definition and the partial transition function ϕ of the automaton, the subscript Sontag is used here. Of course, you could also use a different symbol, e.g., ψ.

[2]$\mathbb{N}_0$ denotes the natural numbers including 0.